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Exploring Water Quality Assessment through AHP and Picture Fuzzy PROMETHEE - II: An In-Depth Investigation

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Abstract

Effective analysis of water quality is crucial for the well-being of ecosystems and human populations, notably in rivers like Thirumanimutharu. This study advances current methodologies by integrating the picture fuzzy PROMETHEE II approach with the analytic hierarchy process (AHP), offering a robust solution to the challenges posed by data ambiguity and imprecision. Born from the fusion of PROMETHEE II with picture fuzzy sets, this innovative method adeptly represents qualitative criteria and linguistic notions inherent in water quality assessment. A practical case study, utilising real water quality data, demonstrates the superior accuracy and durability of the combined picture fuzzy PROMETHEE II and AHP approach compared to traditional methods. These results significantly contribute to our understanding of water quality assessment, providing essential insights for environmental experts and decision-makers. The approach's efficacy is particularly highlighted in the management and conservation of water resources in the Thirumanimutharu river, emphasising the need for advanced methodologies in environmental conservation and resource management. This research underscores the importance of adopting sophisticated techniques for more informed decision-making, ultimately contributing to the water resources and the preservation of ecological balance in the Thirumanimutharu river and similar water bodies.

Keywords: AHP; picture fuzzy PROMETHEE II; water quality; Thirumanimutharu river.

1 Introduction

Decision-making processes have greatly benefited from the application of data-driven methods and methodologies [5]. Techniques for multi-criteria decision making (MCDM) are crucial. However, as the information age and computer technology advance quickly, vast and difficult fuzzy data begin to appear in every aspect of human activity [2].

There were consequently significant issues. The idea of a fuzzy set (FS) was first introduced by American cybernetic expert Zadeh in 1965 [28]. Through the use of membership degrees to represent ambiguous and fuzzy data, a thorough study of fuzzy sets based just on a membership degree parameter has revealed that fuzzy sets struggle to accurately represent the ambiguity that exists in reality. In order to describe fuzzy information, Atanassov [1] developed the idea of intuitionistic fuzzy set (IFS), which are defined by membership, non-membership, and reluctance dimensions. Within the intuitionistic fuzzy framework, a number of scholars have investigated particular models, producing surprising results and insights. This study presents a novel approach to water quality assessment by integrating the picture fuzzy PROMETHEE II method with the Analytic Hierarchy Process (AHP) to address data ambiguity and imprecision [3], with a case study on the Thirumanimutharu river. The combined method offers enhanced accuracy and durability [18], providing valuable insights for environmental decision-making and water resource management [4, 6]. The flaws in the IFS hypothesis start to show out over time. Numerous IFS theory extension forms and applications [7, 9], such as Pythagorean fuzzy set theory [8, 10], intuitionistic cubic fuzzy set theory [11, 12], neutrosophic set theory [13, 14] and intuitionistic cubic fuzzy set theory, have been developed as a result of the diligent work of many academics.

Every neutrosophic set (NS) has the true membership function, the doubtful membership function, and the false membership function. There seem to be numerous applications for NS. Since the function values of the three NS functions are all subsets of the nonstandard unit interval, solving practical problems is tough. Thus, the picture fuzzy set (PFS) [16, 15] and the spherical fuzzy set (SFS) [17, 20] are presented as two important subclasses of NS. They can be distinguished from one another by their membership levels, which include positive, neutral, negative, and refuse. SFS and its applications have expanded quickly in recent years [19, 21]. PFSs and SFSs are primarily differentiated by the fact that the former do not lie inside the range of the standard unit [0, 1] [24, 22]. As a result, PFS is acknowledged as the NS [23, 26]. PFS is a useful advanced fuzzy set for representing many occurrences and events that cannot be handled by other sets like FS and IFS. In order to more precisely detect ambiguous and fuzzy information, PFS is used to evaluate the MCDM data. The MCDM problem cannot be resolved using fuzzy or intuitionistic fuzzy information when picture fuzzy is used. But there is a lot of scholarly interest in the topic of MCDM in picture fuzzy situations [25].

Water quality measurements are crucial for the management and monitoring of aquatic ecosystem health as well as for the supply of safe drinking water to human populations. It is essential to employ cutting-edge techniques and technology in this situation to accurately examine the many aspects that affect water quality. The Thirumanimutharu river's water quality is evaluated in this study using a novel method that combines the picture fuzzy PROMETHEE II method and the AHP method.

Using a combination of the picture fuzzy PROMETHEE II method and the AHP method, the study presents a novel approach to assessing the water quality of the Thirumanimutharu river. This technique is noteworthy for taking a novel approach to solving the problems that fuzzy and ambiguous data in MCDM processes bring. The suggested method improves the accuracy of detecting fuzzy and ambiguous information in MCDM data by introducing picture fuzzy sets (PFS), which provide a more nuanced representation of uncertainty than typical fuzzy and intuitionistic fuzzy sets. This discovery is especially useful in situations where precise analysis is essential to the health of ecosystems and the welfare of humans, such as water quality evaluation. The study also advances the subject of decision-making processes by illustrating how the picture fuzzy PROMETHEE II approach may be applied and is successful in solving MCDM difficulties.

2 Thirumanimutharu River

The Thirumanimutharu river originates from Manjavadi in Shevaroy Mountain and flows through the Salem and Namakkal District. It arises in the Yercaud hills. River Thirumanimutharu joins the River Cauvery at Nanjai Edayar place in Namakkal District. The Thirumanimutharu river, depicted in Figure 1, is a vital source of water for the nearby communities. However, due to increased urbanisation and industrial activity, concerns have been raised about the river's water quality. A detailed and reliable assessment approach is needed to monitor the river's health and find potential contamination sources.

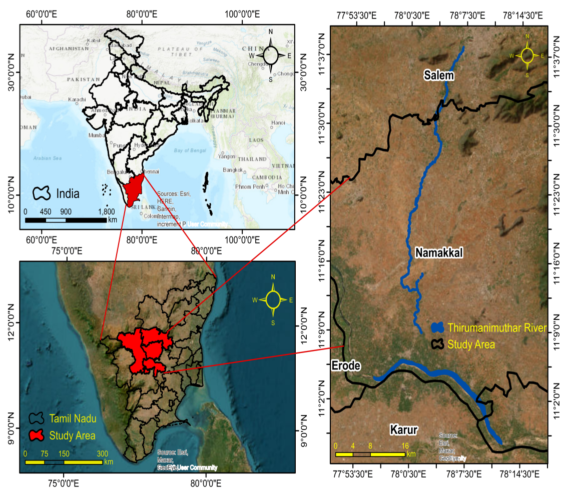

Figure 1: Thirumanimutharu river map.

3 Materials and Methods

3.1 Analytical hierarchy process

In the realm of decision-making, the AHP seems to be a powerful tool, especially when approaching complex situations. Operations researcher and mathematician, Thomas L. Saaty developed AHP in the late 1970s as a structured approach to deal with the challenges that MCDM poses. With a methodical process of prioritisation and comparison, AHP helps decision-makers make well-informed conclusions, which is why it is a cornerstone of rational decision analysis. AHP is a valuable tool for decision-making teams that need to navigate complex decisions and devise organised strategies to handle complexity. Fundamental to AHP is the process of organising choice components into a hierarchical structure, akin to a ranking, as illustrated in

Figure 2. By capturing the relationships between different components, this hierarchy allows decision-makers to carefully weigh the relevance of each one. To facilitate the assessment, matrix comparisons are performed between every possible pair inside every cluster of choice component. Decision-makers can ensure that their decisions are rational and well-founded by using the consistency ratio to assess the internal consistency of their choices and judgements [29]. The usefulness of the results of AHP is improved by giving weights to each component in a cluster according to their respective relevance. You may measure the relative relevance of different components in an organised manner with the use of these weightings.

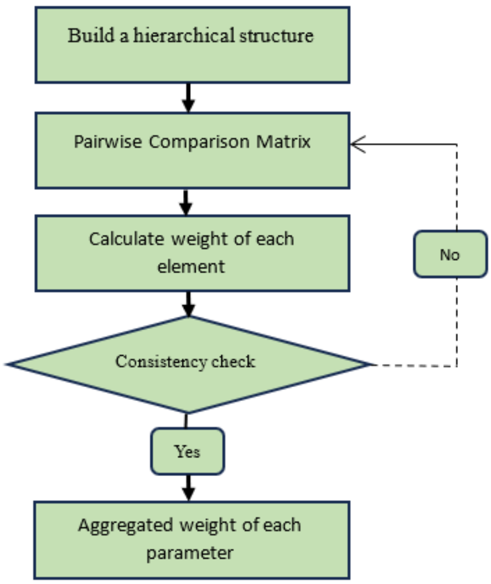

Figure 2: Working rule for AHP.

Step 1: Determining and choosing the decision tree's properties inside a hierarchical framework.

Establishing the study's hierarchy structure is the first step in any AHP analysis (see Figure 3). This hierarchy is essentially a division of a number of attribute levels, each of which stands for a multitude of small, linked sub-groups of characteristics.

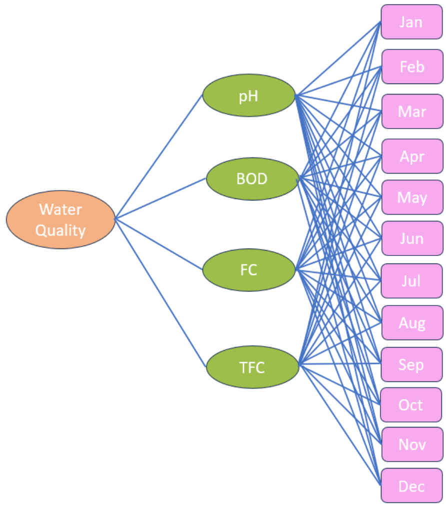

Figure 3: Decision hierarchy.

Step 2: Set up the pairwise comparison matrix.

A matrix of pair-wise comparison (MPC) collects findings of expert and expert ratings. Professional judgements are stated in an MPS analysis of the MADM problem using AHP, where a result. The producer specifies a judgement by inserting the entry a_{ij} where $(a_{ij} > 0)$, indicating how much more important attribute i is than attribute j. The MPC is defined as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}. \tag{1}$$

Step 3: Weighting vectors of attributes to be calculated.

Additive weighting methods are used to account for cardinal numerical values that represent the overall preference of each defined choice. Comparable scores from 1 to 9 were provided by Saaty as a foundation for converting linguistic judgements into cardinal in Table 1.

Table 1: Ratio scale.

Numerical rating	Importance
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values

Step 4: Finding the approximate principal Eigenvector's relative weights.

The process of averaging over the normalised columns involves the computation of characteristic weights. The optimal match for characteristics requires a priority matrix that reflects the estimate of the matrix's eigenvalues. Divide the relative weights of each distinct attribute by the column-sum of the collected weights to achieve the goal of having the weights added up to equal one.

Step 5: Verifying the consistency of the attributes.

The manufacturer might be required to conduct compensation transactions within the characteristic values if the computed disparities are larger than 10. The determined priorities are only legitimate if there are comparable measures that show consistency or near consistency. The approximate stability ratio can be computed using (2):

$$CR = \frac{CI}{RI}. \tag{2}$$

Proceeding with Equation (2), the consistency ratio satisfies with CR < 0.1.

PFSs Intuitionistic fuzzy sets (IFSs) [21] are extended by picture fuzzy sets (PFSs) [28]. PFSs comprise four parameters: the degrees of positive, neutral, negative, and rejection membership. Since PFSs incorporate all relevant evaluation data, including yes, abstention, no, and rejection, they are able to precisely characterise decision-makers' preferences. As a result, the data more closely resembles a real decision-making environment than IFSs [27, 29]. It can also prevent any information loss from the evaluation process.

3.2 Picture fuzzy PROMETHEE II method

The PROMETHEE II technique was modified to use the picture fuzzy PROMETHEE II method for assessing and ranking options depending on a variety of factors. By including picture fuzzy numbers to account for uncertainty and imprecision, this method expands on traditional method PROMETHEE II. In practical decision-making contexts, picture fuzzy numbers offer a more adaptable depiction of uncertainty.

Step 1: Create and standardise the choice matrix.

For the location in the Thirumanimutharu river for the year 2022, experts provide assessment values in the form of picture fuzzy numbers (PFNs) based on four criteria: pH c1, biological oxygen demand (BOD) c2, faecal coliform (FC) c3, and total faecal coliform (TFC) c4. For instance, experts can assign a high, medium, or low rating to the criteria. Moreover, if the expert's knowledge base is limited, they can decide not to provide an assessment value. In essence, the four dimensions of positive, neutral, negative, and refusal memberships inside the PFNs framework are matched by the assessment's high, medium, low, and refusal possibilities. In order to guarantee the accuracy and efficacy of the evaluation data, experts are not allowed to communicate with one another during the assessment process and no matching details are revealed. Due to the PFNs supplied by the experts' membership degrees being equally relevant, it is also possible to achieve the final decision matrix displayed in Table 2 by averaging each membership degree.

Step 2: Calculate the preference index.

With reference to the PROMETHEE II method, the preference index can be calculated as follows:

$$\rho\Big[\Big(\varphi_i, \varphi_k\Big)\Big]. \tag{3}$$

Step 3: Calculate the leaving flow and entering flow.

The leaving flow and entering flow is defined as follows:

$$\gamma^{+}(\phi_i) = \frac{1}{n-1}\sum_{\phi_k}\rho(\phi_i, \phi_k), \tag{4}$$

$$\gamma^{-}(\phi_i) = \frac{1}{n-1}\sum_{\phi_i}\rho(\phi_k, \phi_i). \tag{5}$$

Step 4: Net outranking flow.

The net outranking flow value is obtained from the difference of (4) and (5):

$$\gamma(\phi_i) = \gamma^{+}(\phi_i) - \gamma^{-}(\phi_i). \tag{6}$$

Step 5: Rank the alternatives.

Based on (6), the ranks for the alternatives are ranked accordingly.

The criteria weights are evaluated, and it is in Table 2.

Table 2: Criteria weight.

	pH	BOD	FC	TFC
Criteria Weight	0.2465	0.0667	0.1232	0.5520

Based on the above algorithm, the picture fuzzy decision matrix is framed and it is represented in Table 3.

Table 3: Picture fuzzy decision matrix.

Month	pH	BOD	FC	TFC
Jan	(0.5,0.3,0.15)	(0.64,0.11,0.15)	(0.7,0.13,0.16)	(0.72,0.03,0.16)
Feb	(0.65,0.19,0.11)	(0.7,0.08,0.04)	(0.67,0.13,0.10)	(0.69,0.14,0.09)
Mar	(0.65,0.11,0.21)	(0.66,0.05,0.24)	(0.77,0.05,0.17)	(0.6,0.13,0.20)
Apr	(0.7,0.15,0.14)	(0.65,0.14,0.2)	(0.73,0.03,0.15)	(0.62,0.12,0.2)
May	(0.67,0.13,0.10)	(0.60,0.13,0.20)	(0.7,0.13,0.16)	(0.72,0.03,0.16)
Jun	(0.77,0.05,0.17)	(0.64,0.10,0.12)	(0.66,0.11,0.12)	(0.69.0.10.0.13)
Jul	(0.6,0.3,0.1)	(0.6,0.14,0.21)	(0.74,0.06,0.15)	(0.72,0.07,0.12)
Aug	(0.69,0.1,0.11)	(0.65,0.07,0.23)	(0.73,0.05,0.20)	(0.62,0.12,0.17)
Sep	(0.66,0.17,0.1)	(0.72,0.03,0.16)	(0.65,0.21,0.10)	(0.66,0.11,0.12)
Oct	(0.62,0.2,0.12)	(0.69,0.14,0.09)	(0.6,0.3,0.1)	(0.7,0.13,0.16)
Nov	(0.68,0.10,0.13)	(0.6,0.13,0.20)	(0.69,0.1,0.11)	(0.67,0.13,0.10)
Dec	(0.72,0.07,0.12)	(0.62,0.12,0.2)	(0.66,0.17,0.1)	(0.72,0.15,0.12)

Table 4 shows the leaving and entering flow, while Table 5 indicates the net outranking flow value.

Table 4: Leaving flow and entering flow.

Month	$\gamma^+(\phi_i)$	$\gamma^-(\phi_i)$
Jan	0.0236	0.0465
Feb	0.0254	0.0541
Mar	0.0182	0.0264
Apr	0.0212	0.0325
May	0.0221	0.0341
Jun	0.0245	0.0574
Jul	0.0266	0.0438
Aug	0.0089	0.0224
Sep	0.0192	0.0287
Oct	0.0242	0.0536
Nov	0.0094	0.0424
Dec	0.0263	0.0489

Table 5: Net outranking flow.

Month	$\gamma(\phi_i)$
Jan	-0.0229
Feb	-0.0287
Mar	-0.0082
Apr	-0.0113
May	-0.012
Jun	-0.0329
Jul	-0.0172
Aug	-0.0135
Sep	-0.0095
Oct	-0.0294
Nov	-0.033
Dec	-0.0226

Then, the rank graph of Thirumanimutharu river is illustrated in Figure 4.

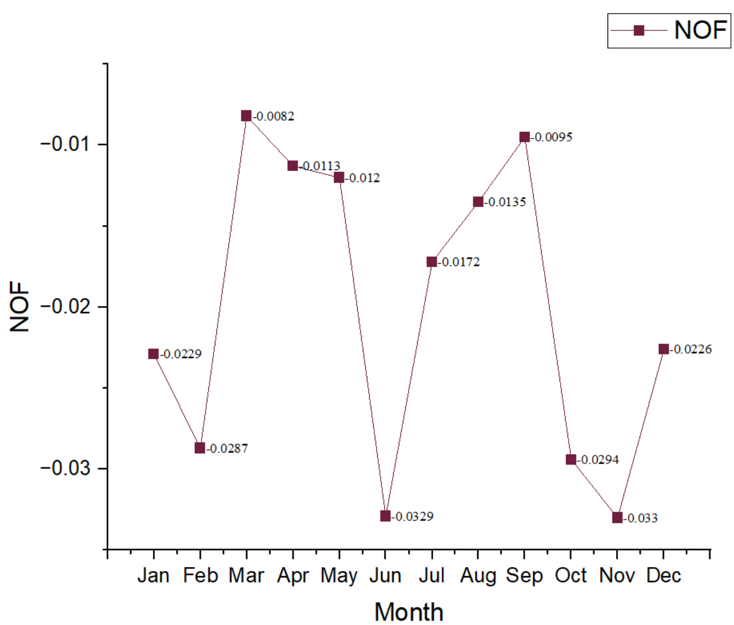

Figure 4: Rank graph of Thirumanimutharu river.

4 Results and Discussions

Two different methodologies have been combined to create an integrated framework for decision-making about water quality. With the help of this framework, unclear data is effectively managed, increasing decision accuracy. Concurrent sensitivity analysis has been performed to validate the viability of our methodology and guarantee its robustness. Our analysis is well-founded since we have meticulously established the parameters for the first fuzzy decision matrix. According to our research, which is shown in Figure 4, the Thirumanimutharu river's water quality is considered fine in November but worrying in March. This is one of the key findings. This thorough approach highlights our dedication to offering insightful and useful evaluations of water quality.

5 Research Implications

The merging of the methodologies, which provides improved capabilities in displaying uncertainty information through positive, neutral, negative, and rejection degrees utilising PFNs, sets the proposed methodology apart from other methods.

These research ramifications stem from the study's primary goal of developing an integrated decision-making methodology for the water quality issue. The first step in this study's methodology is the construction of an assessment index system based on criteria for environmental effect. Second, while some research has offered a number of ways for calculating the water quality index, it is unable to take decision-makers' psychological tendencies into account. Third, the picture fuzzy PROMETHEE II is an effective method for addressing these issues, with enhanced decision-making accuracy. However, it does have some limitations that have been described.

6 Conclusion

In summary, our strategy addresses the complex problem of water quality evaluation by merging the picture-fuzzy PROMETHEE II technique with the AHP in a synergistic integration of decision-making methodologies. Our thorough investigation produces a number of important findings. First of all, our approach deftly tackles the complex problems related to water quality evaluation by deftly integrating decision-makers' viewpoints. Our technique takes into account the inherent ambiguity in data and integrates qualitative inputs by employing the PFS method, hence augmenting the comprehensiveness of our decision-making approach.

Furthermore, the efficacy and accuracy of water quality measures are improved by the integration of these techniques. This integrated method contributes to a greater knowledge of the dynamics of water quality while also improving the accuracy of assessments and offering insightful information. Among the noteworthy conclusions drawn from our research is the detection of seasonal fluctuations in the quality of the water; November showed acceptable circumstances, while March showed unacceptable conditions. This temporal dimension highlights the dynamic character of water quality parameters over various time periods and adds another degree of depth to our research. All things considered, our integrated methodology-which takes stakeholder viewpoints and technical correctness into account-emerges as a solid and perceptive approach to handling the complexity of water quality evaluation.

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Conflicts of Interest The authors declare no conflict of interest.

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